

# Radiation forces on an absorbing micrometer-sized sphere in an evanescent field

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The vertical radiation force on an absorbing micrometer-sized dielectric sphere situated in an evanescent field is calculated, using electromagnetic wave theory. The present work is a continuation of an earlier paper [E. Almaas and I. Brevik, J. Opt. Soc. Am. B **12**, 2429 (1995)], in which both the horizontal and the vertical radiation forces were calculated with the constraint that the sphere was nonabsorbing. Whereas the horizontal force can be well accounted for within this constraint, there is no possibility to describe the *repulsiveness* of the vertical force, so distinctly demonstrated in the Kawata-Sugiura experiment [Opt. Lett. **17**, 772 (1992)], unless a departure from the theory of pure nondispersive dielectrics is made in some way. Introduction of absorption, *i.e.*, a complex refractive index, is one natural way of generalizing the previous theory. We work out general expressions for the vertical force for this case and illustrate the calculations by numerical computations. It turns out that, when applied to the Kawata-Sugiura case, the repulsive radiation force caused by absorption is *not* strong enough to account for the actual lifting of the polystyrene latex or glass spheres. The physical reason for the experimental outcome is, in this case, most probably the presence of surfactants making the surface of the spheres partially conducting.

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## I. INTRODUCTION

The outcome of the Kawata-Sugiura levitation experiment from 1992 [1] is in some ways still surprising. The authors examined the radiation force on a micrometer-sized spherical dielectric particle in the evanescent field produced by a laser beam of moderate power,  $P = 150$  mW. As one would expect, the particle moved along the horizontal plate at a speed of a few micrometers per second, as a result of the tunneled photons in the evanescent field. The surprising point is, however: why does the particle *lift* from the surface? As reported in Ref. [1], “the particle is forced to float from the substrate surface and to slide along the surface.” For a nonmagnetic and non-absorbing dielectric particle, it is well known that the electromagnetic volume force density  $\mathbf{f}$  is

$$\mathbf{f} = -\frac{1}{2} E^2 \nabla \epsilon, \quad (1)$$

$\epsilon$  being the permittivity such that  $\mathbf{f} = 0$  in the homogeneous and isotropic interior, and different from zero only in the particle’s boundary layer. The evanescent field is known to vary as  $\exp[-\beta(x+h)]$  in the vertical direction (see Eq. (10) below), and is significant only very close to the substrate. In practice, this field interacts with those parts of the particle that are situated closest to the substrate, and it gives rise to a surface force that, according to Eq. (1), has to act in the *downward* direction (assuming that  $\epsilon > \epsilon_0$ ). It is simply impossible to account for

a lift force on the particle, using Eq. (1). This discrepancy between electromagnetic theory for dielectrics and the Kawata-Sugiura experiment [1] is therefore, as mentioned above, somewhat surprising at first sight, since Eq. (1) has generally been proven to be invaluable in a multitude of cases in electromagnetism and optics.

Consequently, we have to conclude that the explanation for the observed lift of the particle has to lie in the presence of an additional force, different from the one given by Eq. (1). The following two possibilities come to mind:

(1) The particle has a complex permittivity  $\epsilon$ , and is therefore exposed to a repulsive (or positive lift) force caused by the absorption of radiation in the interior.

(2) For some unspecified reason – *e.g.*, a layer of adsorbed film on the particle’s surface – it may be appropriate to regard the surface as conducting. This allows for an analysis of the problem in terms of a multiple scattering formalism. Adopting such a picture, the vertical force on the particle necessarily has to be repulsive; the photons are bouncing off the spherical surface and are, accordingly, transferring an impulse in the upward direction.

In the present paper, we will be studying option (1) above, thus allowing for a complex  $\epsilon$ , which in turn implies a nonvanishing conductivity  $\sigma$  in the interior of the particle. As far as we know, such a study has not previously been undertaken. This study is a continuation of the work of Almaas and Brevik [2], dealing with the case of a pure dielectric (real  $\epsilon$ ). Whereas the analysis in [2] gave satisfactory results for the *horizontal* radiation force observed in the Kawata-Sugiura experiment, it was impossible, as mentioned above, on the basis of Eq. (1) to account for an upward directed vertical force. From now

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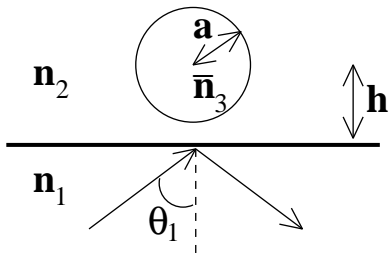


FIG. 1: Spherical particle of radius  $a$ , with complex refractive index  $\bar{n}_3$ , situated in an evanescent field with its center at a height  $h$  above the plane substrate. A laser beam is incident from below at an angle of incidence  $\theta_1 > \theta_{crit}$ . Refractive indices in the transparent media 1 and 2 are respectively  $n_1$  and  $n_2$ .

on, we will assume that the particle is a sphere of radius  $a$ , and regard the conductivity  $\sigma$  as an input parameter.

Before embarking on the mathematical formalism, let us also comment on option (2). Lester and Nieto-Vesperinas [3] recently analyzed the problem using multiple scattering methods, and were able to account for a positive lift force. An advantage of this kind of method is that the proximity of the sphere-substrate two-body system is addressed explicitly. In contrast, our formalism is built on the model of scattering of an evanescent wave by an isolated sphere, and is therefore not expected to be very accurate for small separations between the sphere and the substrate. It is necessary to point out that the adoption of a multiple scattering method does not *in itself* imply that the lift force is positive, as long as the material in the sphere is a pure dielectric. Regardless of whether the electromagnetic boundary conditions at the surface of the substrate are taken into account or not, for a pure dielectric, the force on the sphere is given by Eq. (1), and, as we have argued above, is necessarily downward directed. To account for a positive lift force, we must allow for deviations from the simple theory of pure dielectrics, meaning that absorption, and consequently conductivity, are important effects in the theory, explicitly or implicitly.

In the next section, we summarize the wave theoretical formalism describing the interaction between the evanescent field and the sphere. In Section 3, we show how the surface force  $\mathbf{F}^{surf}$  – *i.e.* the part of the total radiation force  $\mathbf{F}$  which is associated with the boundary and independent of the absorptive properties in the interior – can be written as a double sum over complex expansion coefficients. Section 4, the main section of our paper, contains the calculation of the absorptive part  $\mathbf{F}^{abs}$  of the radiation force. The results are illustrated in Figs. 2 - 5. Our calculations incorporate both the two polarization states of the incident beam ( $p$  and  $s$  polarization).

Our main analytical result is given in Eq. (61), for the nondimensional vertical absorptive force component  $Q_x^{abs}$ . As one would expect physically, the result becomes proportional to the conductivity  $\sigma$ . For the polystyrene

latex spheres, or glass spheres, used in the Kawata-Sugiura experiment, the values of  $\sigma$  are very low. Hence, it turns out that  $Q_x^{abs}$  becomes too weak to account for the floating of the sphere in the gravitational field. Now, our formalism has, of course, a much wider scope than the Kawata-Sugiura experiment. In optics, there are numerous cases where radiation forces interact with absorbing media. For instance, the current development of micro-electromagnetical systems, which are used in a variety of applications including even measurements of the Casimir force, accentuates the usefulness of the present theory.

## II. BASIC FORMALISM

A sketch of the geometry is shown in Fig. 1: A laser beam is incident from below through a transparent medium, called medium 1, and hits the plane, horizontal surface towards the transparent medium 2 at an angle of incidence,  $\theta_1$ , which is greater than the critical angle,  $\theta_{crit} = \arcsin(n_2/n_1)$ , for total reflection. This establishes an evanescent field above the plane surface, which, in turn, is regarded as the incident field falling upon a compact sphere of radius  $a$  and complex refractive index  $\bar{n}_3$ . The sphere is centered at the origin with the  $x$  axis pointing vertically upwards. If  $\bar{\epsilon}_3$  is the complex permittivity of the (nonmagnetic) sphere, it is related to the conductivity,  $\sigma$ , through the equation

$$\bar{\epsilon}_3 = \epsilon_3 + i\sigma/\omega, \quad (2)$$

hence  $\bar{n}_3 = \sqrt{\bar{\epsilon}_3/\epsilon_0}$ . We will assume that the incoming beam in medium 1 is a plane wave, such that its electric and magnetic field amplitudes are related through  $H_0 = \sqrt{\epsilon_0/\mu_0} E_0$ . We introduce the nondimensional wave number,  $\alpha$ , in medium 2, and the relative radial coordinate  $\tilde{r}$ :

$$\alpha = k_2 a = n_2 \omega a / c, \quad \tilde{r} = r / a, \quad (3)$$

and make use of the standard spherical coordinates  $(r, \theta, \varphi)$  centered at the origin of the sphere. Expansions for the incident electric field  $\mathbf{E}^{(i)} = (E_r^{(i)}, E_\theta^{(i)}, E_\varphi^{(i)})$ , the scattered field  $\mathbf{E}^{(s)} = (E_r^{(s)}, E_\theta^{(s)}, E_\varphi^{(s)})$ , and the internal field in the sphere,  $\mathbf{E}^{(w)} = (E_r^{(w)}, E_\theta^{(w)}, E_\varphi^{(w)})$ , are for convenience collected in Appendix A together with the analogous expansions of the magnetic field. In these expressions, we have imposed the electromagnetic boundary conditions at the spherical surface,  $r = a$ . We will need use of the Riccati-Bessel functions  $\psi_l$  and  $\xi_l^{(1)}$ :

$$\psi_l(x) = x j_l(x), \quad \xi_l^{(1)}(x) = x h_l^{(1)}(x). \quad (4)$$

Here,  $j_l$  and  $h_l$  are the spherical Bessel and Hankel functions, satisfying the Wronskian  $W\{\psi_l, \xi_l^{(1)}\} = i$ . There are three groups of expansion coefficients:  $\{A_{lm}, B_{lm}\}$  for the incident field,  $\{a_{lm}, b_{lm}\}$  for the scattered field,

and  $\{c_{lm}, d_{lm}\}$  for the internal field. As for the first-mentioned coefficients, they are determined by inverting the formulas (A.1) and (A.4) for  $E_r^{(i)}$  and  $H_r^{(i)}$ :

$$A_{lm} = \frac{(b/a)^2}{l(l+1)\psi_l(k_2b)} \int_{\Omega} E_r^{(i)}(b, \theta, \varphi) Y_{lm}^*(\theta, \varphi) d\Omega, \quad (5)$$

$$B_{lm} = \frac{(b/a)^2}{H_0 l(l+1)\psi_l(k_2b)} \int_{\Omega} H_r^{(i)}(b, \theta, \varphi) Y_{lm}^*(\theta, \varphi) d\Omega, \quad (6)$$

where  $d\Omega = \sin\theta d\theta d\varphi$ . Here, the angular integrations are taken over a sphere of arbitrary radius  $b$ . This freedom of choice for  $b$  gives us a calculational advantage. The results for  $A_{lm}$  and  $B_{lm}$  are only related to the incident field, and they have to be independent of  $b$ . Calculational insensitivity with respect to various input values for  $b$  thus serves as a check of the evaluations of  $A_{lm}$  and  $B_{lm}$ . In the following, we will suppress the writing of the time factors  $\exp(-i\omega t)$ .

Our expansion procedure follows the conventions of Barton *et. al* [4, 5]. This procedure was also followed in Ref. [2]. In the case where  $b$  is chosen equal to  $a$ , Eqs. (5) and (6) agree with Eqs. (A.23) and (A.24) of Ref. [5].

In order to calculate  $A_{lm}$  and  $B_{lm}$ , it is necessary

to know the incident field components  $E_r^{(i)} \equiv E_r^{(2)}$  and  $H_r^{(2)} \equiv H_r^{(2)}$  in region 2. We introduce the parameters

$$\beta = \frac{n_1\omega}{c} \sqrt{\sin^2\theta_1 - n_{21}^2}, \quad \gamma = \frac{n_1\omega}{c} \sin\theta_1, \quad n_{21} = \frac{n_2}{n_1}, \quad (7)$$

and take the following expressions for the amplitude ratios,  $T_{\parallel} = E_{\parallel}^{(2)}/E_{\parallel}^{(1)}$ , and  $T_{\perp} = E_{\perp}^{(2)}/E_{\perp}^{(1)}$  at the surface of the substrate into account:

$$T_{\parallel} = \frac{2n_{21} \cos\theta_1}{n_{21}^2 \cos\theta_1 + i\sqrt{\sin^2\theta_1 - n_{21}^2}}, \quad (8)$$

$$T_{\perp} = \frac{2 \cos\theta_1}{\cos\theta_1 + i\sqrt{\sin^2\theta_1 - n_{21}^2}}. \quad (9)$$

Here,  $E_{\parallel}$  refers to the field component in the plane of incidence ( $p$  polarization), and  $E_{\perp}$  to the field orthogonal to it ( $s$  polarization). For the radial field components, we find

$$E_r^{(i)} = \left\{ \frac{1}{n_{21}} T_{\parallel} E_{\parallel}^{(1)} [\sin\theta_1 \sin\theta \cos\varphi - i\sqrt{\sin^2\theta_1 - n_{21}^2} \cos\theta] + T_{\perp} E_{\perp}^{(1)} \sin\theta \sin\varphi \right\} \times \exp[-\beta(x+h) + i\gamma z], \quad (10)$$

$$H_r^{(i)} = \left\{ T_{\perp} H_{\parallel}^{(1)} [-\sin\theta_1 \sin\theta \cos\varphi + i\sqrt{\sin^2\theta_1 - n_{21}^2} \cos\theta] + n_{21} T_{\parallel} H_{\perp}^{(1)} \sin\theta \sin\varphi \right\} \times \exp[-\beta(x+h) + i\gamma z], \quad (11)$$

where

$$H_{\parallel}^{(2)}/H_{\parallel}^{(1)} = n_{21} T_{\perp}, \quad H_{\perp}^{(2)}/H_{\perp}^{(1)} = n_{21} T_{\parallel}, \quad (12)$$

at the surface of the substrate. (Readers interested in background material for our developments can *e.g.* consult the articles [2–6], as well as the standard texts [7–9].) Upon insertion into Eqs. (5) and (6), we find that the coefficients  $A_{lm}$  and  $B_{lm}$  for the case of  $s$  ( $\perp$ ) and  $p$  ( $\parallel$ ) polarization are related through

$$A_{lm}^{\perp} = \frac{T_{\perp}}{n_2 T_{\parallel}} B_{lm}^{\parallel}, \quad (13)$$

$$B_{lm}^{\perp} = -\frac{n_2 T_{\perp}}{T_{\parallel}} A_{lm}^{\parallel}, \quad (14)$$

hence, it is sufficient to only consider one polarization direction in the calculations of the expansion coefficients, and in the following, we will choose the case of  $p$  polarization. By inserting the expressions for the incident field into Eqs. (5) and (6), we obtain the following expressions

for the expansion coefficients:

$$A_{lm}^{\parallel} = \frac{\alpha_1(l, m)}{n_{21}} T_{\parallel} e^{-\beta h} \left[ \sin\theta_1 Q_1(l, m) - i\sqrt{\sin^2\theta_1 - n_{21}^2} Q_2(l, m) \right], \quad (15)$$

$$B_{lm}^{\parallel} = n_2 \alpha_1(l, m) T_{\parallel} e^{-\beta h} Q_3(l, m), \quad (16)$$

where we have defined

$$\alpha_1(l, m) = \left[ \frac{2l+1}{4\pi} \frac{(l-m)!}{(l+m)!} \right]^{1/2} \frac{(b/a)^2}{l(l+1)\psi_l(k_2b)}, \quad (17)$$

$$Q_1(l, m) = 2\pi(-1)^{m-1} \int_0^{\pi/2} \sin^2\theta \left\{ \begin{matrix} \cos \\ i \sin \end{matrix} \right\} (\gamma b \cos\theta) \times P_l^m(\cos\theta) [I_{|m-1|}(u) + I_{|m+1|}(u)] d\theta, \quad (18)$$

$$Q_2(l, m) = 4\pi(-1)^m \int_0^{\pi/2} \sin\theta \cos\theta \left\{ \begin{matrix} i \sin \\ \cos \end{matrix} \right\} (\gamma b \cos\theta) \times P_l^m(\cos\theta) I_{|m|}(u) d\theta, \quad (19)$$

$$Q_3(l, m) = 4\pi i(-1)^m \frac{m}{\beta b} \int_0^{\pi/2} \sin \theta \left\{ \begin{array}{c} \cos \\ i \sin \end{array} \right\} (\gamma b \cos \theta) \times P_l^m(\cos \theta) I_{|m|}(u) d\theta, \quad (20)$$

where  $u = \beta b \sin \theta$ ,  $(l + m)$  is  $\begin{cases} \text{even} \\ \text{odd} \end{cases}$ , and  $I_m(z) = i^{-m} J_m(iz)$  is a modified Bessel function. (In Ref. [2], Eqs. (46) and (49), were missing factors of 2.) Again, the calculated values of  $A_{lm}^{\parallel}$  and  $B_{lm}^{\parallel}$  have to be numerically independent of the value chosen for the radius  $b$ .

Once  $A_{lm}$  and  $B_{lm}$  are known, the other sets of coefficients,  $\{a_{lm}, b_{lm}\}$  and  $\{c_{lm}, d_{lm}\}$ , can readily be calculated using Eqs. (A.19)–(A.22). In appendix B, we give a table of  $A_{lm}$  and  $B_{lm}$  up to  $l_{\max} = 7$  for  $p$  polarization.

### III. THE SURFACE FORCE

Let us, as indicated above, write the total radiation force  $\mathbf{F}$  on the sphere as a sum of two contributions:

$$\mathbf{F} = \mathbf{F}^{\text{surf}} + \mathbf{F}^{\text{abs}}, \quad (21)$$

where  $\mathbf{F}^{\text{surf}}$  is the result of the force density  $\mathbf{f}$  acting in the surface layer, *i.e.* Eq. (1), and  $\mathbf{F}^{\text{abs}}$  is caused by the absorption in the sphere. By integrating across the surface layer, the components  $F_i^{\text{surf}}$  of  $\mathbf{F}^{\text{surf}}$  can be related to the surface integral of Maxwell's stress tensor  $S_{ik}$  on the outside of the sphere:

$$F_i^{\text{surf}} = - \int S_{ik} n_k dS, \quad (22)$$

$\mathbf{n}$  being the outward normal (cf. for instance, Ref. [10]). Using the general expressions for the incident and the scattered field, Eqs. (A.1) – (A.12), we can re-write this surface integral in terms of the complex expansion coefficients  $A_{lm}$ ,  $B_{lm}$ ,  $a_{lm}$  and  $b_{lm}$ :

$$\begin{aligned} \frac{F_x^{\text{surf}} + i F_y^{\text{surf}}}{\epsilon_0 E_0^2 a^2} &= \frac{i\alpha^2}{4} \sum_{l=1}^{\infty} \sum_{m=-l}^l \left\{ \left[ \frac{(l+m+2)(l+m+1)}{(2l+1)(2l+3)} \right]^{1/2} l(l+2) \left( 2n_2^2 a_{lm} a_{l+1,m+1}^* + n_2^2 a_{lm} A_{l+1,m+1}^* \right. \right. \\ &\quad \left. \left. + n_2^2 A_{lm} a_{l+1,m+1}^* + 2b_{lm} b_{l+1,m+1}^* + b_{lm} B_{l+1,m+1}^* + B_{lm} b_{l+1,m+1}^* \right) + \left[ \frac{(l-m+1)(l-m+2)}{(2l+1)(2l+3)} \right]^{1/2} \right. \\ &\quad \times l(l+2) \left( 2n_2^2 a_{l+1,m-1} a_{lm}^* + n_2^2 a_{l+1,m-1} A_{lm}^* + n_2^2 A_{l+1,m-1} a_{lm}^* + 2b_{l+1,m-1} b_{lm}^* + b_{l+1,m-1} B_{lm}^* \right. \\ &\quad \left. \left. + B_{l+1,m-1} b_{lm}^* \right) - \left[ \frac{(l+m+1)(l-m)}{(2l+1)(2l+3)} \right]^{1/2} n_2 \left( -2a_{lm} b_{l,m+1}^* + 2b_{lm} a_{l,m+1}^* - a_{lm} B_{l,m+1}^* \right. \right. \\ &\quad \left. \left. + b_{lm} A_{l,m+1}^* + B_{lm} a_{l,m+1}^* - A_{lm} b_{l,m+1}^* \right) \right\}, \quad (23) \end{aligned}$$

$$\begin{aligned} \frac{F_z^{\text{surf}}}{\epsilon_0 E_0^2 a^2} &= -\frac{\alpha^2}{2} \sum_{l=1}^{\infty} \sum_{m=-l}^l \left[ \frac{(l-m+1)(l+m+1)}{(2l+1)(2l+3)} \right]^{1/2} l(l+2) \Im \left[ 2n_2^2 a_{l+1,m} a_{lm}^* + n_2^2 a_{l+1,m} A_{lm}^* + n_2^2 A_{l+1,m} a_{lm}^* \right. \\ &\quad \left. + 2b_{l+1,m} b_{lm}^* + b_{l+1,m} B_{lm}^* + B_{l+1,m} b_{lm}^* + n_2 m (2a_{lm} b_{lm}^* + a_{lm} B_{lm}^* + A_{lm} b_{lm}^*) \right]. \quad (24) \end{aligned}$$

We shall not dwell much on these expressions, since our main interest, as mentioned above, is the vertical force  $F_x^{\text{abs}}$  associated with the absorption. Consider, however, the horizontal surface force component  $F_z^{\text{surf}}$ . With an incident beam power (in vacuum) of  $P = 150$  mW, distributed over a circular cross-sectional area of diameter  $10 \mu\text{m}$ , we find the magnitude of the Poynting vector to be  $(c/2)\epsilon_0 E_0^2 = 19.0 \text{ MW/m}^2$ . Taking the radius of

the sphere  $a = 1 \mu\text{m}$ , we find the denominator in Eq. (24) to be  $\epsilon_0 E_0^2 a^2 = 0.13 \text{ pN}$ . If the sphere is made of glass ( $n_3 = 1.50$ ) and the surrounding medium 2 is water ( $n_2 = 1.33$ ), we then have  $\alpha = 2\pi a/\lambda_2 = 7.9$ , since the wavelength in water for Nd:YAG laser light is  $\lambda_2 = 1.06/1.33 = 0.80 \mu\text{m}$ . Using Fig. 5 in Ref. [2], we find that  $F_z^{\text{surf}}$  is positive, as it should be, and is approximately equal to  $0.010 \text{ pN}$  in the case of  $p$  polarization.

Since the Reynolds number is very low, the Stokes drag formula  $D = 6\pi\mu av$  is applicable, using  $\mu = 1.0 \times 10^{-3}$  Pa s as the dynamic viscosity of water. Putting  $F_z^{\text{surf}} = D$ , we obtain the drift velocity of the sphere to be  $v = 0.53 \mu$  m/s. This is an order of magnitude agreement with the observations of Kawata and Sugiura [1]; from their Fig. 4, one infers that  $v \sim 1 - 2 \mu$  m. We consider this agreement to be satisfactory, and think that there is no real need to carry out a complicated calculation to find the absorptive correction to the horizontal radiation force. In the following, we will therefore focus our attention solely on the *vertical* absorptive part of the force.

#### IV. THE VERTICAL ABSORPTIVE FORCE

To calculate the absorptive force, we first replace Eq. (1) by the general expression

$$\mathbf{f} = \rho \mathbf{E} + \mathbf{J} \times \mathbf{B} - \frac{1}{2} E^2 \nabla \epsilon, \quad (25)$$

where  $\rho$  and  $\mathbf{J}$  are the external charge and current densities in the medium. Assuming that no external charges are present,  $\rho = 0$ . Writing  $\mathbf{J} = \sigma \mathbf{E}$  with  $\sigma$  as the constant conductivity, we get, in complex representation, the following expression for the absorptive force:

$$\mathbf{F}^{\text{abs}} = \frac{1}{2} \Re \int \mathbf{J} \times \mathbf{B}^* dV = \frac{\sigma \mu_0}{2} \Re \int \mathbf{E} \times \mathbf{H}^* dV, \quad (26)$$

where the integral is evaluated over the volume of the sphere.

Exploiting the following property, we can simplify our calculation: In cases of current interest, the conductivity,  $\sigma$ , of the material in the sphere is a small quantity. Therefore, when evaluating the lowest order contribution from absorption, we may replace the fields inside the sphere by those calculated when assuming the material to be a perfect dielectric. That is, we replace the complex refractive index  $\bar{n}_3$  by its real part  $n_3$ . This facilitates the calculation of the volume integrals. The expressions for the internal fields are given by Eqs. (A.13)–(A.18), in spherical coordinates. For notational convenience, we will omit the superscripts ( $w$ ).

With  $\tilde{r} = r/a$  we can write the vertical absorptive force as

$$\begin{aligned} F_x^{\text{abs}} &= \frac{\sigma \mu_0}{2} a^3 \Re \int_0^{2\pi} d\varphi \int_0^\pi \sin \theta d\theta \int_0^1 \tilde{r}^2 d\tilde{r} \left[ (\mathbf{E} \times \mathbf{H}^*)_r \right. \\ &\quad \left. \times \sin \theta \cos \varphi + (\mathbf{E} \times \mathbf{H}^*)_\theta \cos \theta \cos \varphi - (\mathbf{E} \times \mathbf{H}^*)_\varphi \sin \varphi \right], \\ &= \frac{\sigma \mu_0}{2} a^3 \Re \sum_{i=1}^6 I_i, \end{aligned} \quad (27)$$

where we have defined

$$I_1 = \int_0^{2\pi} \cos \varphi d\varphi \int_0^\pi \sin^2 \theta d\theta \int_0^1 \tilde{r}^2 d\tilde{r} E_\theta H_\varphi^*, \quad (28)$$

$$I_2 = - \int_0^{2\pi} \cos \varphi d\varphi \int_0^\pi \sin^2 \theta d\theta \int_0^1 \tilde{r}^2 d\tilde{r} E_\varphi H_\theta^*, \quad (29)$$

$$I_3 = \int_0^{2\pi} \cos \varphi d\varphi \int_0^\pi \cos \theta \sin \theta d\theta \int_0^1 \tilde{r}^2 d\tilde{r} E_\varphi H_r^*, \quad (30)$$

$$I_4 = - \int_0^{2\pi} \cos \varphi d\varphi \int_0^\pi \cos \theta \sin \theta d\theta \int_0^1 \tilde{r}^2 d\tilde{r} E_r H_\varphi^*, \quad (31)$$

$$I_5 = - \int_0^{2\pi} \sin \varphi d\varphi \int_0^\pi \sin \theta d\theta \int_0^1 \tilde{r}^2 d\tilde{r} E_r H_\theta^*, \quad (32)$$

$$I_6 = \int_0^{2\pi} \sin \varphi d\varphi \int_0^\pi \sin \theta d\theta \int_0^1 \tilde{r}^2 d\tilde{r} E_\theta H_r^*. \quad (33)$$

It is now convenient to introduce the operators

$$\mathcal{L} = \sum_{l=1}^{\infty} \sum_{j=1}^{\infty} \int_0^{n_{32}\alpha} du, \quad (34)$$

$$\mathcal{M}_{lj}^{(\pm)} = \sum_{m=-l}^l \sum_{k=-j}^j (\delta_{m,k-1} \pm \delta_{m,k+1}). \quad (35)$$

We write the two first of the  $I_i$ 's as a sum of four terms:

$$I_i = i\pi\alpha n_1 c \epsilon_0 E_0^2 \sum_{k=1}^4 I_{ik}, \quad i = 1, 2. \quad (36)$$

and the remaining  $I_i$ 's, which are simpler since they are each composed of only two terms, as

$$I_i = i\pi\alpha n_1 c \epsilon_0 E_0^2 \sum_{k=1}^2 I_{ik}, \quad i = 3, 4, 5, 6. \quad (37)$$

where

$$I_{11} = -n_{32} \mathcal{L} \left[ \psi'_l(u) \psi'_j(u) \mathcal{M}_{lj}^{(+)} \left\{ k c_{lm} d_{jk}^* C_{lm} C_{jk} \mathcal{R}_1(l, m, j, k) \right\} \right], \quad (38)$$

$$I_{12} = -n_{32}^2 n_2 \mathcal{L} \left[ \psi'_l(u) \psi_j(u) \mathcal{M}_{lj}^{(+)} \left\{ c_{lm} c_{jk}^* C_{lm} C_{jk} \mathcal{R}_2(l, m, j, k) \right\} \right], \quad (39)$$

$$I_{13} = \frac{1}{n_2} \mathcal{L} \left[ \psi_l(u) \psi'_j(u) \mathcal{M}_{lj}^{(+)} \left\{ m k d_{lm} d_{jk}^* C_{lm} C_{jk} \mathcal{R}_3(l, m, j, k) \right\} \right], \quad (40)$$

$$I_{14} = n_{32} \mathcal{L} \left[ \psi_l(u) \psi_j(u) \mathcal{M}_{lj}^{(+)} \left\{ m d_{lm} c_{jk}^* C_{lm} C_{jk} \mathcal{R}_1(j, k, l, m) \right\} \right], \quad (41)$$

$$I_{21} = -n_{32} \mathcal{L} \left[ \psi'_l(u) \psi'_j(u) \mathcal{M}_{lj}^{(+)} \left\{ m c_{lm} d_{jk}^* C_{lm} C_{jk} \mathcal{R}_1(j, k, l, m) \right\} \right], \quad (42)$$

$$I_{22} = -n_{32}^2 \mathcal{L} \left[ \psi'_l(u) \psi_j(u) \mathcal{M}_{lj}^{(+)} \left\{ m k c_{lm} c_{jk}^* C_{lm} C_{jk} \mathcal{R}_3(l, m, j, k) \right\} \right], \quad (43)$$

$$I_{23} = \frac{1}{n_2} \mathcal{L} \left[ \psi_l(u) \psi'_j(u) \mathcal{M}_{lj}^{(+)} \left\{ d_{lm} d_{jk}^* C_{lm} C_{jk} \mathcal{R}_2(l, m, j, k) \right\} \right], \quad (44)$$

$$I_{24} = n_{32} \mathcal{L} \left[ \psi_l(u) \psi_j(u) \mathcal{M}_{lj}^{(+)} \left\{ k d_{lm} c_{jk}^* C_{lm} C_{jk} \mathcal{R}_1(l, m, j, k) \right\} \right], \quad (45)$$

$$I_{31} = n_{32} \mathcal{L} \left[ \frac{\psi'_l(u)}{u} \psi_j(u) j(j+1) \mathcal{M}_{lj}^{(+)} \left\{ m c_{lm} d_{jk}^* C_{lm} C_{jk} \mathcal{R}_4(l, m, j, k) \right\} \right], \quad (46)$$

$$I_{32} = -\frac{1}{n_2} \mathcal{L} \left[ \frac{\psi_l(u)}{u} \psi_j(u) j(j+1) \mathcal{M}_{lj}^{(+)} \left\{ d_{lm} d_{jk}^* C_{lm} C_{jk} \mathcal{R}_5(l, m, j, k) \right\} \right], \quad (47)$$

$$I_{41} = n_{32} \mathcal{L} \left[ \frac{\psi_l(u)}{u} \psi'_j(u) l(l+1) \mathcal{M}_{lj}^{(+)} \left\{ k c_{lm} d_{jk}^* C_{lm} C_{jk} \mathcal{R}_4(l, m, j, k) \right\} \right], \quad (48)$$

$$I_{42} = n_{32}^2 n_2 \mathcal{L} \left[ \frac{\psi_l(u)}{u} \psi_j(u) l(l+1) \mathcal{M}_{lj}^{(+)} \left\{ c_{lm} c_{jk}^* C_{lm} C_{jk} \mathcal{R}_5(j, k, l, m) \right\} \right], \quad (49)$$

$$I_{51} = -n_{32} \mathcal{L} \left[ \frac{\psi_l(u)}{u} \psi'_j(u) l(l+1) \mathcal{M}_{lj}^{(-)} \left\{ c_{lm} d_{jk}^* C_{lm} C_{jk} \mathcal{R}_1(j, k, l, m) \right\} \right], \quad (50)$$

$$I_{52} = -n_{32}^2 n_2 \mathcal{L} \left[ \frac{\psi_l(u)}{u} \psi_j(u) l(l+1) \mathcal{M}_{lj}^{(-)} \left\{ k c_{lm} c_{jk}^* C_{lm} C_{jk} \mathcal{R}_3(l, m, j, k) \right\} \right], \quad (51)$$

$$I_{61} = n_{32} \mathcal{L} \left[ \frac{\psi'_l(u)}{u} \psi_j(u) j(j+1) \mathcal{M}_{lj}^{(-)} \left\{ c_{lm} d_{jk}^* C_{lm} C_{jk} \mathcal{R}_1(l, m, j, k) \right\} \right], \quad (52)$$

$$I_{62} = -\frac{1}{n_2} \mathcal{L} \left[ \frac{\psi_l(u)}{u} \psi_j(u) j(j+1) \mathcal{M}_{lj}^{(-)} \left\{ m d_{lm} d_{jk}^* C_{lm} C_{jk} \mathcal{R}_3(l, m, j, k) \right\} \right]. \quad (53)$$

with

$$\mathcal{R}_3(l, m, j, k) = \int_0^\pi P_l^m(\cos \theta) P_j^k(\cos \theta) d\theta, \quad (56)$$

$$\mathcal{R}_1(l, m, j, k) = \int_0^\pi \sin \theta \frac{dP_l^m(\cos \theta)}{d\theta} P_j^k(\cos \theta) d\theta, \quad (54)$$

$$\mathcal{R}_4(l, m, j, k) = \int_0^\pi \cos \theta P_l^m(\cos \theta) P_j^k(\cos \theta) d\theta, \quad (57)$$

$$\mathcal{R}_2(l, m, j, k) = \int_0^\pi \sin^2 \theta \frac{dP_l^m(\cos \theta)}{d\theta} \frac{dP_j^k(\cos \theta)}{d\theta} d\theta, \quad (55)$$

$$\mathcal{R}_5(l, m, j, k) = \int_0^\pi \cos \theta \sin \theta \frac{dP_l^m(\cos \theta)}{d\theta} P_j^k(\cos \theta) d\theta, \quad (58)$$

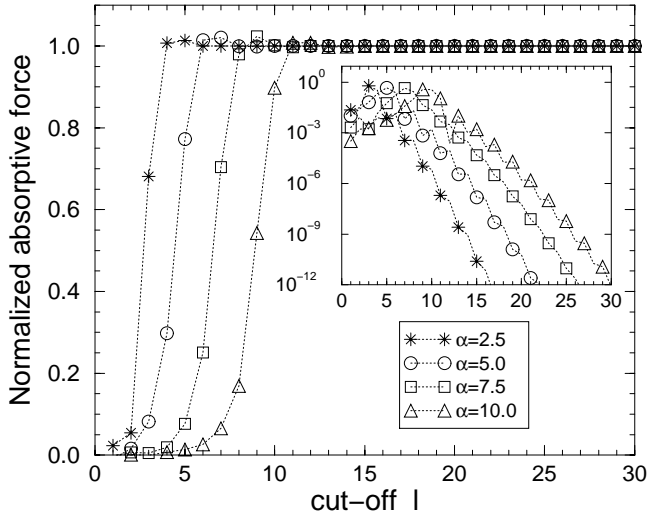


FIG. 2: Plot of the total absorptive force versus increasing cut-off in Eq. (34) for 4 values of  $\alpha$ : 2.5, 5.0, 7.5, and 10.0, with  $n_1 = 1.75$ ,  $n_2 = 1.33$ , and  $n_3 = 1.50$ . The inset shows the absolute value of the difference in the total force using  $l$  and  $(l-1)$  respectively, as cut-offs.

and the definition

$$C_{lm} = \sqrt{\frac{(2l+1)(l-m)!}{4\pi(l+m)!}}. \quad (59)$$

Altogether, writing the nondimensional vertical absorptive force as  $Q_x^{\text{abs}}$ , where

$$Q_x^{\text{abs}} = \frac{F_x^{\text{abs}}}{\epsilon_0 E_0^2 a^2}, \quad (60)$$

we get

$$Q_x^{\text{abs}} = -\sigma \mu_0 c \lambda \frac{n_{12} \alpha^2}{4} \Im \left[ \sum_{i=1}^2 \sum_{k=1}^4 I_{ik} + \sum_{i=3}^6 \sum_{k=1}^2 I_{ik} \right]. \quad (61)$$

Note that the prefactor in the above equations, as well as the terms  $I_{ik}$ , are nondimensional. The expression (61) is our main result.

## V. RESULTS

### A. Numerics

We implemented an adaptive Gauss-Kronrod rule to calculate the integrals in Eqs. (18) – (20) and (38) – (53). By employing the symmetry properties of the integrals over the Legendre polynomials (Eqs. (54) – (58)), we were able to greatly reduce the number of integrals to be computed. To make sure that the sum over  $l$  and  $j$  in Eq. (34) converged properly for our chosen cut-off value, we plotted the total absorptive force on the sphere versus

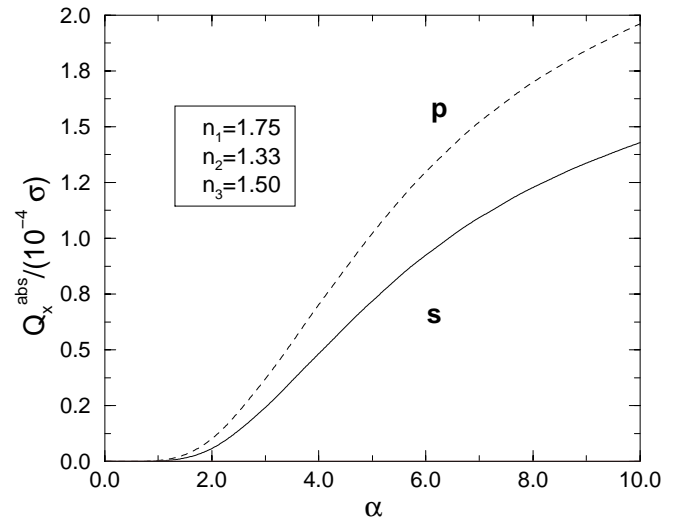


FIG. 3: Nondimensional vertical absorptive force  $Q_x^{\text{abs}} = F_x^{\text{abs}}/(\epsilon_0 E_0^2 a^2)$  versus particle size parameter  $\alpha = k_2 a$  for the set of refractive indices shown. The two states of polarization for the incident beam are distinguished. Here, as well as in the subsequent figures,  $\theta_1 = 51^\circ$ . Also, all figures refer to sphere resting on the plate, *i.e.* to  $h = a$ . The figure is scaled against the conductivity  $\sigma$  of the sphere.

increasing cut-off,  $l_{\text{max}}$ , in  $l$  (we used the same cut-off for  $l$  and  $j$ ). In Fig. 2, we give an example for  $p$  polarization using several values of  $\alpha$ . The total force is normalized to the  $l_{\text{max}} = 30$  result; all other parameters are given in the figure caption. This figure clearly suggests that, for a given value of  $\alpha$ , there is a narrow range in  $l$  which accounts for most of the absorptive force. Moreover, it is seen that this range of important  $l$ -values moves to higher  $l$  and broadens with increasing  $\alpha$ . In the inset, we plot on a linear-log scale the absolute value of the difference in the total force using  $l$  and  $(l-1)$  respectively, as cut-offs. The inset indicates that contributions to the absorptive force from large  $l$ -values (large with respect to the  $l$ -value of the peak) are exponentially suppressed. Hence, for the range of values for  $\alpha$ ,  $n_1$ ,  $n_2$ , and  $n_3$  studied in this article, we need only consider contributions to the force from  $l \leq 30$ .

It is of interest to point out that the main features of Fig. 2 can be understood from a physical point of view. (This kind of argument is frequently made use of in connection with Casimir-related calculations; cf. for instance, Ref. [11]). The most significant angular momenta are those that are of the same order of magnitude as, or are somewhat smaller than, the angular momentum corresponding to grazing incidence on the sphere. Consider now a photon with energy  $\hbar\omega$  and momentum  $\hbar k_2$  that just touches the surface of the sphere. Its angular momentum is  $\hbar k_2 a$ . Setting the angular momentum equal to  $\hbar l$ , we thus obtain  $l = k_2 a = \alpha$ . Thus, the most significant values of  $l$  should be expected to be  $l \lesssim \alpha$ . This is seen to be in accordance with the calculated re-

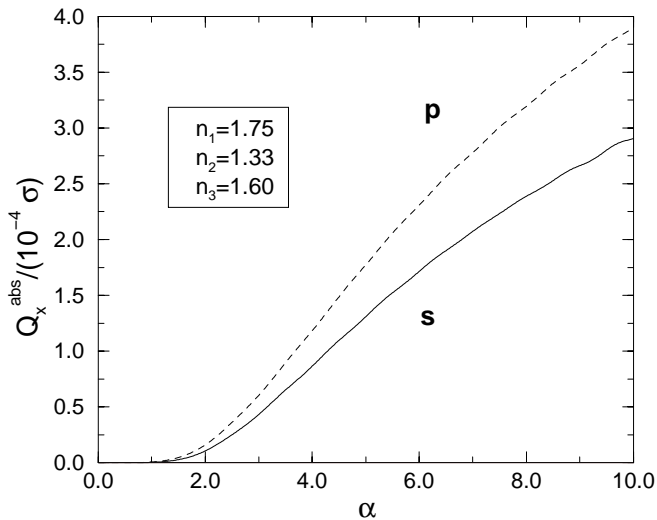


FIG. 4: Same as for Fig. 3, but with a higher (dominant real part of) refractive index ( $n_3 = 1.60$ ) in the sphere.

sults in Fig. 2. In view of the crudeness of the argument, the agreement is actually better than we had reason to expect.

### B. Discussion of the figures

Illustrative examples of the behavior of the nondimensional vertical absorptive force are shown in Figs. 3 – 5. The angle of incidence is  $\theta_1 = 51^\circ$  everywhere. In the case of  $p$  polarization,  $E_0 = E_{\parallel}^{(1)}$ , whereas for  $s$  polarization,  $E_0 = E_{\perp}^{(1)}$ . When the incident field is taken to be a plane wave, the amplitudes  $E_{\parallel}^{(1)}$  and  $E_{\perp}^{(1)}$  are constants. It is to be kept in mind that, in order to lift the sphere from the plate, the positive absorptive force has to overcome not only gravity, but also the negative surface force  $F_x^{\text{surf}}$  that was calculated in Ref. [2] for the same values of  $\{n_1, n_2, n_3\}$  as we consider here. The cases treated in the two papers are thus directly comparable.

Similarly to Ref. [2], all curves in Figs. 3 – 5 refer to the case of contact between sphere and plate, *i.e.* to  $h = a$ . From Eqs. (15) and (16), it is seen that  $A_{lm}$  and  $B_{lm}$  contain  $\exp(-\beta h)$  as a common factor. The same property is carried over to the coefficients  $c_{lm}$  and  $d_{lm}$ , according to Eqs. (A.21) and (A.22). As the absorptive force is quadratic in the last-mentioned coefficients, it follows that  $Q_x^{\text{abs}}$  at an arbitrary height  $h$  is equal to  $\exp[-2\beta(h-a)]$  times the value that can be read off from Figs. 3 – 5. No figures need to be worked out to show the dependence of the force with height above the plate.

In order to interpret the figures physically, it is helpful to consider a concrete example. Let us relate our discussion essentially to the situation considered in Section 3: incident plane wave of power  $P = 150$  mW, corresponding to  $\epsilon_0 E_0^2 a^2 = 0.13$  pN when the radius of the sphere is

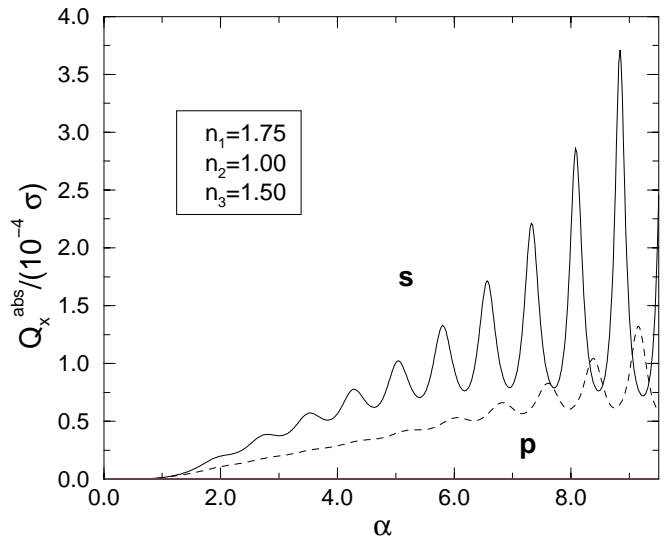


FIG. 5: Same as for Fig. 3, but with vacuum ( $n_2 = 1$ ) surrounding the sphere.

$a = 1 \mu\text{m}$ . If the density of the sphere is  $2.4 \text{ g/cm}^3$  (the same density as for glass), the weight of it is  $mg = 0.10$  pN. Moreover,  $n_3 = 1.50$ ,  $n_2 = 1.33$  (water), whereas the size parameter is  $\alpha = 7.9$  for Nd:YAG laser light. Let us look for the necessary value of  $\sigma$  in order to satisfy the condition for elevation of the sphere, just above the plate. The condition is  $F_x^{\text{abs}} = mg + |F_x^{\text{surf}}|$ . Expressed nondimensionally,

$$Q_x^{\text{abs}} = \frac{mg}{\epsilon_0 E_0^2 a^2} + |Q_x^{\text{surf}}|. \quad (62)$$

From Fig. 3 we read off, when choosing for definiteness the case of  $p$  polarization,  $Q_x^{\text{abs}}/(10^{-4}\sigma) = 1.7$ , whereas from Fig. 4 in [2] we read off  $Q_x^{\text{surf}} = -0.35$  for the same value of  $\alpha$ . This leads to a relatively high value of  $\sigma$ , about  $6600 \text{ S/m}$  ( $S \equiv \Omega^{-1}$ ). In our example, the influence of the weight of the sphere is relatively large because the incident power is so moderate. If we increase  $P$  to 1 watt, we get  $\epsilon_0 E_0^2 a^2 = 0.87$  pN, resulting in a somewhat lower value,  $\sigma \approx 2700 \text{ S/m}$ .

For glass,  $\sigma \approx 10^{-12} \text{ S/m}$ . We can thus immediately conclude that absorption is *not* the physical reason for the elevation of the sphere in the Kawata-Sugiura experiment. It is likely that there were surfactants on the spheres in this experiment, and hence, that a theoretical description of the kind given in Ref. [3] is most appropriate in this case. For other materials where the conductivities are higher, for instance for carbon ( $\sigma = 0.77 \times 10^5 \text{ S/m}$ ), the absorption-generated elevation of microspheres should be quite possible physically. One must however be aware of the algebraic restriction made in the present paper to simplify calculations: our calculation relied on the property that  $\sigma/\epsilon_0\omega \ll 1$  (cf. Eq. (2)). This restriction made it possible to take the refractive index  $n_3$  to be real, inside the integrals in Section 4. Since  $\epsilon_0\omega \approx 10^5$



S/m, we can no longer expect high accuracy from the formalism when  $\sigma \gg 10^4$  S/m.

Consider finally the remaining figures: Fig. 4 shows that when the refractive index in the sphere increases from 1.50 to 1.60, the absorptive force increases. This is as we would expect. Moreover, if the difference between interior and exterior indices increases, the Riccati-Bessel functions cause the force to become more oscillatory in character. The changes are clearly shown, if we compare Fig. 3 with Fig. 5. Increasing differences between the refractive indices effectively mean enhancing the geometrical-optics properties of the system. There is also a dependence of the force upon the two different polarization states of the beam. No simple physical explanation of this dependence seems to exist.

## VI. CONCLUSIONS

We can summarize as follows:

1. The strategy of the above calculation has been to identify the evanescent field with the incident field on the sphere. The theory makes use of wave optics, and is thus applicable even when the wavelength is of the same magnitude as the sphere diameter.

2. The main novel development of the present paper in comparison with Ref. [2] is the calculation in Section 4 of the vertical absorptive force, assuming the refractive index  $\bar{n}_3$  in the sphere to be complex. The calculation rests upon the approximation that  $\sigma/\epsilon_0\omega \ll 1$ , so that the fields inside the sphere can be evaluated as if the material were a perfect dielectric.

3. The results of Figs. 3 – 5 show that in order to account for elevation of the micrometer-sphere, the con-

ductivity  $\sigma$  must be at least as large as about  $10^3$  S/m. This applies, for instance, to media like carbon.

4. For latex or glass spheres, used in the Kawata-Sugiura experiment [1], the absorptive force is too weak to account for the elevation force. Most probably, the observed elevation force was due to a film on the surface of the sphere, making it partly conducting. Cf. also Ref. [3].

## VII. ACKNOWLEDGEMENTS

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## APPENDIX A: EXPANSIONS OF THE ELECTROMAGNETIC FIELDS

For convenience, we summarize the expansions for the incident, (*i*), the scattered, (*s*), and the internal, (*w*), field components. As mentioned in the main text, subscript 1 refers to the nonabsorbing substrate, subscript 2 to the nonabsorbing medium around the sphere (refractive index  $n_2$ ), and subscript 3 to the absorbing sphere whose complex refractive index is  $\bar{n}_3$ . The relative refractive index is  $\bar{n}_{32} = \bar{n}_3/n_2$ . Furthermore,  $E_0$  and  $H_0$  refer to the incoming plane wave in the substrate, the nondimensional wave number and distance is  $\alpha = k_2 a = n_2 \omega a / c$ , and  $\tilde{r} = r/a$  respectively.

---

*Incident field*

$$E_r^{(i)} = \frac{E_0}{\tilde{r}^2} \sum_{l=1}^{\infty} \sum_{m=-l}^l l(l+1) A_{lm} \psi_l(\alpha \tilde{r}) Y_{lm}, \quad (\text{A.1})$$

$$E_{\theta}^{(i)} = \frac{\alpha E_0}{\tilde{r}} \sum_{l=1}^{\infty} \sum_{m=-l}^l \left[ A_{lm} \psi'_l(\alpha \tilde{r}) \frac{\partial Y_{lm}}{\partial \theta} - \frac{m}{n_2} B_{lm} \psi_l(\alpha \tilde{r}) \frac{Y_{lm}}{\sin \theta} \right], \quad (\text{A.2})$$

$$E_{\varphi}^{(i)} = \frac{\alpha E_0}{\tilde{r}} \sum_{l=1}^{\infty} \sum_{m=-l}^l \left[ i m A_{lm} \psi'_l(\alpha \tilde{r}) \frac{Y_{lm}}{\sin \theta} - \frac{i}{n_2} B_{lm} \psi_l(\alpha \tilde{r}) \frac{\partial Y_{lm}}{\partial \theta} \right], \quad (\text{A.3})$$

$$H_r^{(i)} = \frac{H_0}{\tilde{r}^2} \sum_{l=1}^{\infty} \sum_{m=-l}^l l(l+1) B_{lm} \psi_l(\alpha \tilde{r}) Y_{lm}, \quad (\text{A.4})$$

$$H_{\theta}^{(i)} = \frac{\alpha H_0}{\tilde{r}} \sum_{l=1}^{\infty} \sum_{m=-l}^l \left[ B_{lm} \psi'_l(\alpha \tilde{r}) \frac{\partial Y_{lm}}{\partial \theta} + m n_2 A_{lm} \psi_l(\alpha \tilde{r}) \frac{Y_{lm}}{\sin \theta} \right], \quad (\text{A.5})$$

$$H_{\varphi}^{(i)} = \frac{\alpha H_0}{\tilde{r}} \sum_{l=1}^{\infty} \sum_{m=-l}^l \left[ i m B_{lm} \psi'_l(\alpha \tilde{r}) \frac{Y_{lm}}{\sin \theta} + i n_2 A_{lm} \psi_l(\alpha \tilde{r}) \frac{\partial Y_{lm}}{\partial \theta} \right]. \quad (\text{A.6})$$

### Scattered field

$$E_r^{(s)} = \frac{E_0}{\tilde{r}^2} \sum_{l=1}^{\infty} \sum_{m=-l}^l l(l+1) a_{lm} \xi_l^{(1)}(\alpha \tilde{r}) Y_{lm}, \quad (\text{A.7})$$

$$E_{\theta}^{(s)} = \frac{\alpha E_0}{\tilde{r}} \sum_{l=1}^{\infty} \sum_{m=-l}^l \left[ a_{lm} \xi_l^{(1)'}(\alpha \tilde{r}) \frac{\partial Y_{lm}}{\partial \theta} - \frac{m}{n_2} b_{lm} \xi_l^{(1)}(\alpha \tilde{r}) \frac{Y_{lm}}{\sin \theta} \right], \quad (\text{A.8})$$

$$E_{\varphi}^{(s)} = \frac{\alpha E_0}{\tilde{r}} \sum_{l=1}^{\infty} \sum_{m=-l}^l \left[ i m a_{lm} \xi_l^{(1)'}(\alpha \tilde{r}) \frac{Y_{lm}}{\sin \theta} - \frac{i}{n_2} b_{lm} \xi_l^{(1)}(\alpha \tilde{r}) \frac{\partial Y_{lm}}{\partial \theta} \right], \quad (\text{A.9})$$

$$H_r^{(s)} = \frac{H_0}{\tilde{r}^2} \sum_{l=1}^{\infty} \sum_{m=-l}^l l(l+1) b_{lm} \xi_l^{(1)}(\alpha \tilde{r}) Y_{lm}, \quad (\text{A.10})$$

$$H_{\theta}^{(s)} = \frac{\alpha H_0}{\tilde{r}} \sum_{l=1}^{\infty} \sum_{m=-l}^l \left[ b_{lm} \xi_l^{(1)'}(\alpha \tilde{r}) \frac{\partial Y_{lm}}{\partial \theta} + m n_2 a_{lm} \xi_l^{(1)}(\alpha \tilde{r}) \frac{Y_{lm}}{\sin \theta} \right], \quad (\text{A.11})$$

$$H_{\varphi}^{(s)} = \frac{\alpha H_0}{\tilde{r}} \sum_{l=1}^{\infty} \sum_{m=-l}^l \left[ i m b_{lm} \xi_l^{(1)'}(\alpha \tilde{r}) \frac{Y_{lm}}{\sin \theta} + i n_2 a_{lm} \xi_l^{(1)}(\alpha \tilde{r}) \frac{\partial Y_{lm}}{\partial \theta} \right]. \quad (\text{A.12})$$

### Internal field

$$E_r^{(w)} = \frac{E_0}{\tilde{r}^2} \sum_{l=1}^{\infty} \sum_{m=-l}^l l(l+1) c_{lm} \psi_l(\bar{n}_{32} \alpha \tilde{r}) Y_{lm}, \quad (\text{A.13})$$

$$E_{\theta}^{(w)} = \frac{\alpha E_0}{\tilde{r}} \sum_{l=1}^{\infty} \sum_{m=-l}^l \left[ \bar{n}_{32} c_{lm} \psi_l'(\bar{n}_{32} \alpha \tilde{r}) \frac{\partial Y_{lm}}{\partial \theta} - \frac{m}{n_2} d_{lm} \psi_l(\bar{n}_{32} \alpha \tilde{r}) \frac{Y_{lm}}{\sin \theta} \right], \quad (\text{A.14})$$

$$E_{\varphi}^{(w)} = \frac{\alpha E_0}{\tilde{r}} \sum_{l=1}^{\infty} \sum_{m=-l}^l \left[ i m \bar{n}_{32} c_{lm} \psi_l'(\bar{n}_{32} \alpha \tilde{r}) \frac{Y_{lm}}{\sin \theta} - \frac{i}{n_2} d_{lm} \psi_l(\bar{n}_{32} \alpha \tilde{r}) \frac{\partial Y_{lm}}{\partial \theta} \right], \quad (\text{A.15})$$

$$H_r^{(w)} = \frac{H_0}{\tilde{r}^2} \sum_{l=1}^{\infty} \sum_{m=-l}^l l(l+1) d_{lm} \psi_l(\bar{n}_{32} \alpha \tilde{r}) Y_{lm}, \quad (\text{A.16})$$

$$H_{\theta}^{(w)} = \frac{\alpha H_0}{\tilde{r}} \sum_{l=1}^{\infty} \sum_{m=-l}^l \left[ \bar{n}_{32} d_{lm} \psi_l'(\bar{n}_{32} \alpha \tilde{r}) \frac{\partial Y_{lm}}{\partial \theta} + m n_2 \bar{n}_{32}^2 c_{lm} \psi_l(\bar{n}_{32} \alpha \tilde{r}) \frac{Y_{lm}}{\sin \theta} \right], \quad (\text{A.17})$$

$$H_{\varphi}^{(w)} = \frac{\alpha H_0}{\tilde{r}} \sum_{l=1}^{\infty} \sum_{m=-l}^l \left[ i m \bar{n}_{32} d_{lm} \psi_l'(\bar{n}_{32} \alpha \tilde{r}) \frac{Y_{lm}}{\sin \theta} + i n_2 \bar{n}_{32}^2 c_{lm} \psi_l(\bar{n}_{32} \alpha \tilde{r}) \frac{\partial Y_{lm}}{\partial \theta} \right]. \quad (\text{A.18})$$

Everywhere, primes denote differentiation with respect to the whole argument.

### Relations between coefficients

$$a_{lm} = \frac{\psi_l'(\bar{n}_{32} \alpha) \psi_l(\alpha) - \bar{n}_{32} \psi_l(\bar{n}_{32} \alpha) \psi_l'(\alpha)}{\bar{n}_{32} \psi_l(\bar{n}_{32} \alpha) \xi_l^{(1)'}(\alpha) - \psi_l'(\bar{n}_{32} \alpha) \xi_l^{(1)}(\alpha)} A_{lm}, \quad (\text{A.19})$$

$$b_{lm} = \frac{\bar{n}_{32} \psi_l'(\bar{n}_{32} \alpha) \psi_l(\alpha) - \psi_l(\bar{n}_{32} \alpha) \psi_l'(\alpha)}{\psi_l(\bar{n}_{32} \alpha) \xi_l^{(1)'}(\alpha) - \bar{n}_{32} \psi_l'(\bar{n}_{32} \alpha) \xi_l^{(1)}(\alpha)} B_{lm}, \quad (\text{A.20})$$

$$c_{lm} = \frac{i}{\bar{n}_{32}^2 \psi_l(\bar{n}_{32} \alpha) \xi_l^{(1)'}(\alpha) - \bar{n}_{32} \psi_l'(\bar{n}_{32} \alpha) \xi_l^{(1)}(\alpha)} A_{lm}, \quad (\text{A.21})$$

$$d_{lm} = \frac{i}{\psi_l(\bar{n}_{32} \alpha) \xi_l^{(1)'}(\alpha) - \bar{n}_{32} \psi_l'(\bar{n}_{32} \alpha) \xi_l^{(1)}(\alpha)} B_{lm}. \quad (\text{A.22})$$

Here the Wronskian is  $W\{\psi_l, \xi_l^{(1)}\} = i$ .

# APPENDIX B: TABLE OF $A_{lm}$ AND $B_{lm}$

We include a table of the numerical values for the real and imaginary parts of  $A_{lm}$  and  $B_{lm}$  for an evanescent field. The coefficients are calculated for  $p$  polarization, and we have set  $a = 1$  and  $\epsilon^{-\beta h} = 1$  in Eqs. (15) and (16). For space considerations, we only give the coefficients up to  $l = 7$ .

$l$	$m$	$\Re\{A_{lm}\}$	$\Im\{A_{lm}\}$	$\Re\{B_{lm}\}$	$\Im\{B_{lm}\}$
1	-1	7.8951e-14	-3.5261e-14	4.5863e-14	1.0269e-13
1	0	-1.0417e-14	-2.3324e-14	0.0	0.0
1	1	-7.8951e-14	3.5261e-14	4.5863e-14	1.0269e-13
2	-2	-1.2570e-14	5.6142e-15	-7.3021e-15	-1.6350e-14
2	-1	2.8048e-14	6.2800e-14	-7.8266e-14	3.4955e-14
2	0	3.0791e-14	-1.3752e-14	0.0	0.0
2	1	-2.8048e-14	-6.2800e-14	-7.8266e-14	3.4955e-14
2	2	-1.2570e-14	5.6142e-15	7.3021e-15	1.6350e-14
3	-3	2.1752e-15	-9.7150e-16	1.2636e-15	2.8292e-15
3	-2	-7.7601e-15	-1.7375e-14	2.2116e-14	-9.8777e-15
3	-1	-5.7659e-14	2.5752e-14	-3.0232e-14	-6.7690e-14
3	0	1.6820e-14	3.7661e-14	0.0	0.0
3	1	5.7659e-14	-2.5752e-14	-3.0232e-14	-6.7690e-14
3	2	-7.7601e-15	-1.7375e-14	-2.2116e-14	9.8777e-15
3	3	-2.1752e-15	9.7150e-16	1.2636e-15	2.8292e-15
4	-4	-3.9427e-16	1.7609e-16	-2.2903e-16	-5.1281e-16
4	-3	1.8142e-15	4.0620e-15	-5.2075e-15	2.3258e-15
4	-2	2.1681e-14	-9.6834e-15	1.1989e-14	2.6843e-14
4	-1	-2.5411e-14	-5.6895e-14	6.2106e-14	-2.7738e-14
4	0	-4.4609e-14	1.9923e-14	0.0	0.0
4	1	2.5411e-14	5.6895e-14	6.2106e-14	-2.7738e-14
4	2	2.1681e-14	-9.6834e-15	-1.1989e-14	-2.6843e-14
4	3	-1.8142e-15	-4.0620e-15	-5.2075e-15	2.3258e-15
4	4	-3.9427e-16	1.7609e-16	2.2903e-16	5.1281e-16
5	-5	7.3610e-17	-3.2876e-17	4.2760e-17	9.5741e-17
5	-4	-4.0248e-16	-9.0116e-16	1.1595e-15	-5.1784e-16
5	-3	-6.2653e-15	2.7982e-15	-3.5248e-15	-7.8921e-15

$l$	$m$	$\Re\{A_{lm}\}$	$\Im\{A_{lm}\}$	$\Re\{B_{lm}\}$	$\Im\{B_{lm}\}$
5	-2	1.1689e-14	2.6172e-14	-3.1349e-14	1.4001e-14
5	-1	5.8803e-14	-2.6263e-14	2.6420e-14	5.9156e-14
5	0	-2.3242e-14	-5.2039e-14	0.0	0.0
5	1	-5.8803e-14	2.6263e-14	2.6420e-14	5.9156e-14
5	2	1.1689e-14	2.6172e-14	3.1349e-14	-1.4001e-14
5	3	6.2653e-15	-2.7982e-15	-3.5248e-15	-7.8921e-15
5	4	-4.0248e-16	-9.0116e-16	-1.1595e-15	5.1784e-16
5	5	-7.3610e-17	3.2876e-17	4.2760e-17	9.5741e-17
6	-6	-1.4028e-17	6.2652e-18	-8.1489e-18	-1.8246e-17
6	-5	8.7334e-17	1.9554e-16	-2.5213e-16	1.1261e-16
6	-4	1.6276e-15	-7.2694e-16	9.2343e-16	2.0676e-15
6	-3	-3.9706e-15	-8.8903e-15	1.0972e-14	-4.9002e-15
6	-2	-3.1171e-14	1.3922e-14	-1.6060e-14	-3.5958e-14
6	-1	2.8039e-14	6.2781e-14	-5.7903e-14	2.5861e-14
6	0	6.0270e-14	-2.6918e-14	0.0	0.0
6	1	-2.8039e-14	-6.2781e-14	-5.7903e-14	2.5861e-14
6	2	-3.1171e-14	1.3922e-14	1.6060e-14	-3.5958e-14
6	3	3.9706e-15	8.8903e-15	1.0972e-14	-4.9002e-15
6	4	1.6276e-15	-7.2694e-16	-9.2343e-16	-2.0676e-15
6	5	-8.7334e-17	-1.9554e-16	-2.5213e-16	1.1261e-16
6	6	-1.4028e-17	6.2652e-18	8.1489e-18	1.8246e-17
7	-7	2.7140e-18	-1.2121e-18	1.5766e-18	3.5300e-18
7	-6	-1.8745e-17	-4.1970e-17	5.4194e-17	-2.4204e-17
7	-5	-3.9999e-16	1.7865e-16	-2.2806e-16	-5.1063e-16
7	-4	1.1736e-15	2.6278e-15	-3.2899e-15	1.4693e-15
7	-3	1.2042e-14	-5.3781e-15	6.4860e-15	1.4522e-14
7	-2	-1.6489e-14	-3.6919e-14	4.0869e-14	-1.8253e-14
7	-1	-6.8637e-14	3.0655e-14	-2.5861e-14	-5.7905e-14
7	0	3.1083e-14	6.9595e-14	0.0	0.0
7	1	6.8637e-14	-3.0655e-14	-2.5861e-14	-5.7905e-14
7	2	-1.6489e-14	-3.6919e-14	4.0869e-14	1.8253e-14
7	3	-1.2042e-14	5.3781e-15	6.4860e-15	1.4522e-14
7	4	1.1736e-15	2.6278e-15	-3.2899e-15	-1.4693e-15
7	5	3.9999e-16	-1.7865e-16	-2.2806e-16	-5.1063e-16
7	6	-1.8745e-17	-4.1970e-17	5.4194e-17	2.4204e-17
7	7	-2.7140e-18	1.2121e-18	1.5766e-18	3.5300e-18

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